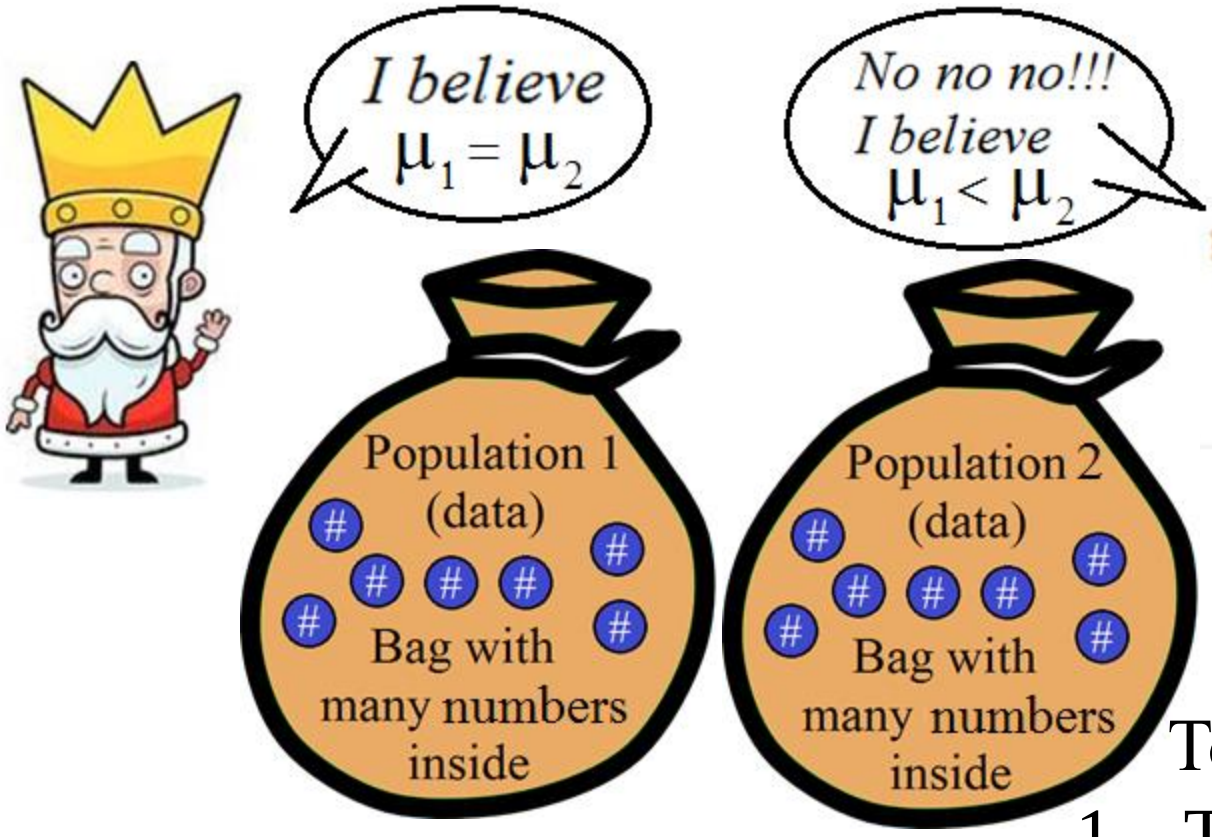
Sec. 11.3: Hypothesis Tests for 2 Population Means μ_1 & μ_2 (independent samples)

Sec. 11.2: Hypothesis Tests for 2 Population Means μ_1 & μ_2 → μ_d (dependent samples)

Idea Behind 2 Population Hypothesis Tests (for μ_1 & μ_2)



independent samples

To settle the dispute…

1. Take 2 samples (one from each population)
2. Calculate $\bar{x}_1$ & $\bar{x}_2$

If $\bar{x}_1 = \bar{x}_2$, believe the king

If $\bar{x}_1 > \bar{x}_2$, believe the king

If $\bar{x}_1 < \bar{x}_2$, it depends!

If $\bar{x}_1 < \bar{x}_2$ but $\bar{x}_1$ is close to $\bar{x}_2$, still believe the king

If $\bar{x}_1 < \bar{x}_2$ but $\bar{x}_1$ is far from $\bar{x}_2$, then believe the peasant

Independent Samples vs. Dependent Samples

Determine if the individuals in one sample were used to determine the individuals in the other sample

If no → Independent Samples

If yes → Dependent Sample

Independent Samples vs. Dependent Samples

Independent Samples

- If the individuals selected for one sample do not dictate which individuals are selected for the second samples
- The 2 samples are in no way related to each other
- In all problems in this class, if the 2 sample sizes are different, they must be independent
- Sometimes if the sample sizes are the same, the samples can still be independent

Independent Samples vs. Dependent Samples

Dependent (Matched-Pairs) Samples

- If the individuals selected for one sample dictate which individuals are selected for the second samples
- The 2 samples are very closely related to each other
- If the samples are dependent, they will have the same sample size (but not the other way around)

Independent Samples vs. Dependent Samples Examples

Example: Are these samples independent or dependent?

Sample 1 = 35 randomly selected PCC students

Sample 2 = 31 randomly selected Rio Hondo students

Answer: Independent

Independent Samples vs. Dependent Samples Examples

Example: Are these samples independent or dependent?

Sample 1 = 18 randomly selected straight married American Men

Sample 2 = The 18 women that are the wives of the men from sample 1

Answer: Dependent

Independent Samples vs. Dependent Samples Examples

Example: Are these samples independent or dependent?

Sample 1 = 50 randomly selected Americans

Sample 2 = The same 50 randomly selected Americans

Answer: Dependent

Independent Samples vs. Dependent Samples Examples

Example: Are these samples independent or dependent?

Sample 1 = 20 randomly selected Americans

Sample 2 = 20 randomly selected Europeans

Answer: Independent

Independent Samples vs. Dependent Samples Examples

Example: Are these samples independent or dependent?

Sample 1 = The 900 Rio Hondo students who enrolled in math 30 this semester

Sample 2 = The 300 Rio Hondo students who enrolled in math 33 this semester

Answer: Independent

Independent Samples vs. Dependent Samples Examples

Example: Are these samples independent or dependent?

Sample 1 = 75 randomly selected kids between the ages of 10 and 18 that have exactly one sibling

Sample 2 = The siblings of the kids in sample 1

Answer: Dependent

Hypothesis Tests for 2 Population Means
μ_1 & μ_2 Formulas & Info (independent samples)

Population 1	Population 2
Population parameter: μ_1	Population parameter: μ_2
Sample 1	Sample 2
Sample size: n_1	Sample size: n_2
Sample mean: $\bar{x}_1$	Sample mean: $\bar{x}_2$
Sample standard deviation: s_1	Sample standard deviation: s_2

Hypothesis Tests for 2 Population Means
μ_1 & μ_2 Formulas & Info (independent samples)

Quantities you are performing a hypothesis test for: μ_1 & μ_2

Probability distribution: *t* - distribution

Degrees of freedom: $df =$ smaller of $n_1 - 1$ & $n_2 - 1$

Test statistic formula (Welch's Approximate t):

- $$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}}$$

(helps you determine how many steps apart $\bar{x}_1 - \bar{x}_2$ and $\mu_1 - \mu_2$ are)

Condition: Population from which samples are drawn have a NORMAL distribution OR $n_1 \geq 30$ & $n_2 \geq 30$

Ex 1 (Sec. 11.3 Book Example 1 pg. 556 independent samples): In the Spacelab Life Sciences 2 payload, 14 male rats were sent to space. Upon their return, the red blood cell mass (in milliliters) of the rats was determined. A control group of 14 male rats was held under the same conditions (except for space flight) as the space rats, and their red blood cell mass was also determined when the space rats returned. The project, led by Dr. Paul X. Callahan, resulted in the data below. Does the evidence suggest that the flight animals have a different red blood cell mass from the control animals at the $\alpha = 0.05$ level of significance?

a) Use the P-value method

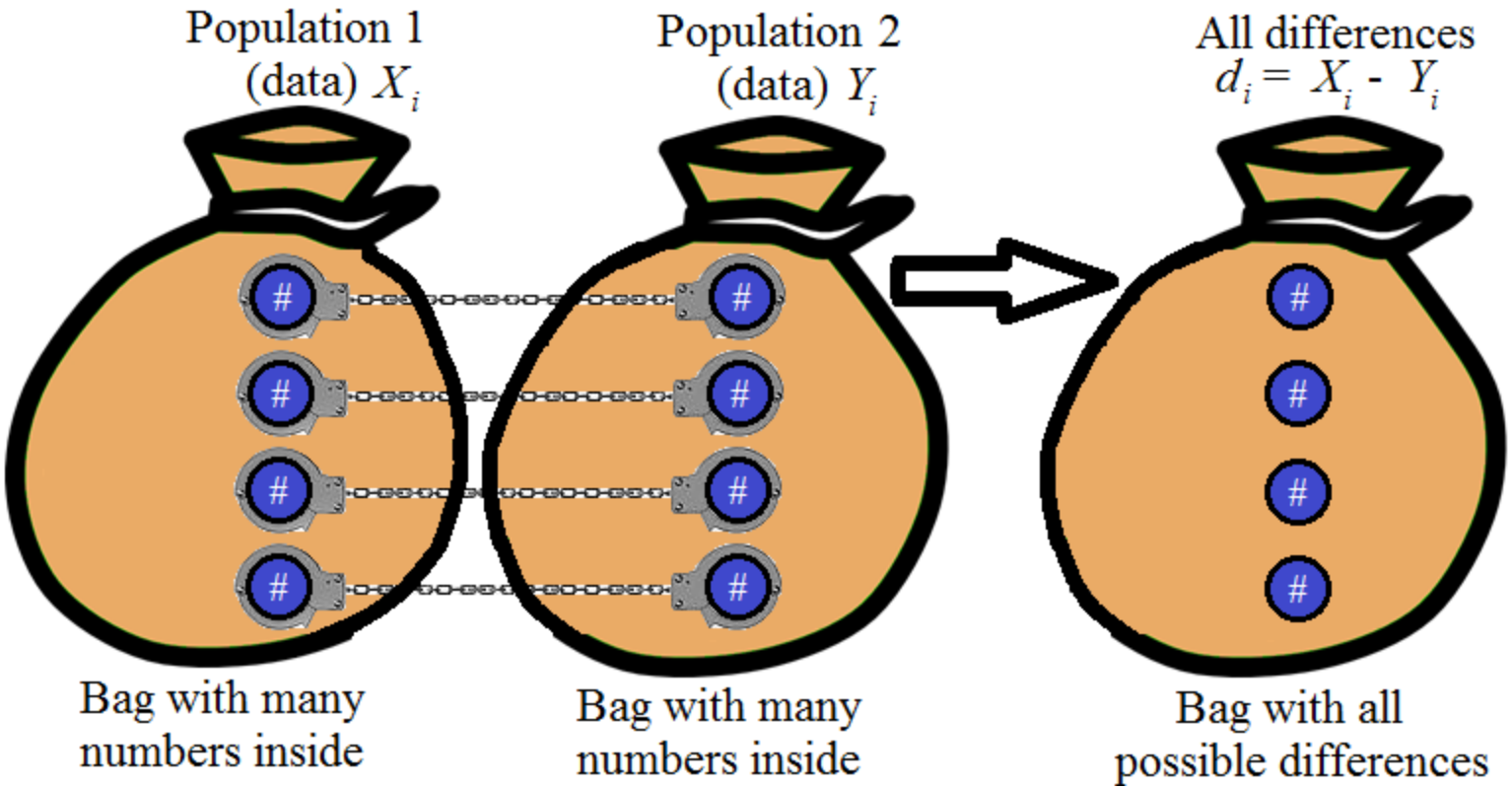
b) Use the rejection region method

Ex 1 (Sec. 11.3 Book Example 1 pg. 556 independent samples):

Flight Rat Red Blood Cell Mass (mL)				Control Rat Red Blood Cell Mass (mL)			
8.59	8.64	7.43	7.21	8.65	6.99	8.40	9.66
6.87	7.89	9.79	6.85	7.62	7.44	8.55	8.70
7.00	8.80	9.30	8.03	7.33	8.58	9.88	9.94
6.39	7.54			7.14	9.14		
		n1 =				n2 =	
		xbar1 =				xbar2 =	
		s1 =				s2 =	

Hypothesis Tests for 2 Population Means μ_1 & μ_2 Formulas & Info (dependent samples)

What's the difference? Differences d_i



Turn a 2 population problem into a 1 population problem by looking at the differences d_i

Hypothesis Tests for 2 Population Means μ_1 & μ_2 Formulas & Info (dependent samples)

Population 1 data notation: X_i

Population 2 data notation: Y_i

Population of differences data notation: $d_i = X_i - Y_i$

Quantity we are performing a hypothesis test for: μ_d

Mean (average) of sample differences: $\bar{d}$

Standard deviation of sample differences: s_d

Hypothesis Tests for 2 Population Means μ_1 & μ_2 Formulas & Info (dependent samples)

Notes:

- H_0 will always be $H_0: \mu_d = 0$
- H_1 will be one of $H_1: \mu_d < 0$, $H_1: \mu_d > 0$, or $H_1: \mu_d \neq 0$
- If H_1 is $H_1: \mu_d < 0$ or $H_1: \mu_d > 0$, how do you decide which one is correct for your problem?
  - Decide which one is population 1, which one is population 2, decide which order you want to subtract in, then figure out what the meaning of a negative answer is and what the meaning of a positive answer is

Hypothesis Tests for 2 Population Means μ_1 & μ_2 Formulas & Info (dependent samples)

Probability distribution: *t* - distribution

Degrees of freedom: $df = n - 1$

(*n* is the common sample size, or the number of differences)

Test statistic formula: $$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

Condition: The differences d_i are normally distributed

or $n \geq 30$

Ex 2 (Sec. 11.2 Book Ex.1, pg. 545 dependent samples):
Professor Andy Neill measured the time (in seconds) required to catch a falling meter stick for 12 randomly selected students' dominant hand and nondominant hand. Professor Neill wants to know if the reaction time in an individual's dominant hand is less than the reaction time in his or her nondominant hand. A coin flip is used to determine whether reaction time is measured using the dominant or nondominant hand first. Conduct the test at the $\alpha = 0.05$ level of significance. The data is on the next slide.

a) Use the P-value method

b) Use the rejection region method

Ex 2 (Sec. 11.2 Book Ex.1, pg. 545 dependent samples):

Student	Dominant hand, Xi	Nondominant hand, Yi	Differences, Xi-Yi
1	0.177	0.179	0.177 - 0.179 = -0.002
2	0.210	0.202	0.210 - 0.202 = 0.008
3	0.186	0.208	-0.022
4	0.189	0.184	0.005
5	0.198	0.215	-0.017
6	0.194	0.193	0.001
7	0.160	0.194	-0.034
8	0.163	0.160	0.003
9	0.166	0.209	-0.043
10	0.152	0.164	-0.012
11	0.190	0.210	-0.020
12	0.172	0.197	-0.025
Source: Professor Andy Neill, Joliet Junior College			